Effect of Thermal Noise on the dc Josephson Effect

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An approximate analytic solution of the Fokker-Planck equation describing the effect of thermal fluctuations on the dc Josephson voltage which is valid for finite capacitance of the junction is obtained. It is shown that our solution should be applicable in the important region of experimental interest.

I. INTRODUCTION

To investigate the effect of thermal fluctuations on the dc Josephson effect, Anderson and Goldman[1] have recently reported very interesting measurements of current-voltage characteristics of a Josephson junction at temperatures sufficiently close to the transition temperature. The junction is kept in series with a large effective resistance and a battery, so that it is driven by a constant (capacitive loading[2]) current source I. They analyze their results by comparing them with a calculation due to Ambegaokar and Halperin[3] which uses an analogy with Brownian motion of a particle in a field of force. A similar analogy was used by Ivanchenko and Zil'berman[4,5] to study the effect of thermal fluctuations. The calculation due to Ambegaokar and Halperin is valid only in the limit of zero capacitance C of the junction, since they solve the Smoluchowski equation instead of the full Fokker-Planck equation, whereas Ivanchenko and Zil'berman give interesting new results[5] only for large capacitance, in the region where $I<I_1$, where I_1 is the "maximum" Josephson current. However, experimental[1] values of the capacitance[6] are such that none of these calculations are applicable, and it may be misleading to fit experimental results with any of these theories.

In this paper we examine the resulting Fokker-Planck equation in the presence of thermal fluctuations and a finite capacitance in a greater detail in order to derive a suitable expression which is valid in the important region of experimental interest. We show that it is possible to obtain a simple analytic expression for the voltage due to thermal noise for $I<I_1$ (more precisely, I has to be less than the cutoff current[2] in the absence of thermal fluctuations, which is always less than I_1 for a finite capacitance), $kT \ll \hbar I_1/e$ and arbitrary C. We also give exact alternative analytic expressions for the case of zero capacitance which are probably simpler to handle.

II. FOKKER-PLANCK EQUATION

For the capacitive loading, the equations satisfied by θ, the phase difference of the order parameter on opposite sides of the junction, and V, the potential difference, are[2,3]

$$\frac{d\theta}{dt}=\frac{2eV}{\hbar}, \tag{1}$$

$$C\,\frac{dV}{dt}=I-I_1\sin\theta-\frac{V}{R}+\tilde{L}(t)\ , \tag{2}$$

where $\tilde{L}(t)$ is the fluctuating thermal noise current, with

$$\langle\tilde{L}(t)\rangle=0,\ \ \langle\tilde{L}(t'+t)\tilde{L}(t')\rangle=(2kT/R)\delta(t)\ . \tag{3}$$

In terms of the momentum variable $p=(\hbar C/2e)V$, and the periodic coordinate θ, this problem is equivalent to the Brownian motion of a particle in the potential $U(\theta)=-(\hbar/2e)(I\theta+I_1\cos\theta)$. Let us introduce dimensionless quantities

$$x=\frac{I}{I_1},\quad v=\frac{V}{I_1R},\quad \mathscr{I}=\frac{\tilde{L}}{I_1},\quad u=\frac{2U(\theta)}{\gamma kT}=-(x\theta+\cos\theta)\ ,$$
$$\gamma=\frac{\hbar I_1}{ekT},\quad \beta_c=\left(\frac{2eI_1}{\hbar C}\right)R^2C^2,\quad \tau=\frac{t}{RC}\ . \tag{4}$$

The equations of motion then become

$$\frac{d\theta}{d\tau}=\beta_c v\ , \tag{5}$$

$$\frac{dv}{d\tau}=(x-\sin\theta)-v+\mathscr{I}(\tau)\ , \tag{6}$$

where

$$\langle\mathscr{I}(\tau'+\tau)\mathscr{I}(\tau')\rangle=(4/\beta_c\gamma)\,\delta(\tau)\ . \tag{7}$$

In the absence of terms on the right-hand side of Eq. (6) which describe the force, damping, and thermal fluctuations, the distribution function $P(v,\theta,\tau)$ satisfies the Liouville equation

$$\frac{\partial P}{\partial\tau}+\frac{d\theta}{d\tau}\,\frac{\partial P}{\partial\theta}=0\ . \tag{8}$$

However, in the presence of the thermal fluctuations and other terms in Eq. (6), if one keeps terms only up to the second moment in Δv, one has the Fokker-Planck equation[7]

$$\frac{\partial P}{\partial \tau}+\frac{d\theta}{d\tau}\frac{\partial P}{\partial \theta}=-\frac{\partial}{\partial v}[A(v)P]+\frac{1}{2}\frac{\partial^2}{\partial v^2}[B(v)P], \quad (9)$$

where

$$A(v)=\lim_{\Delta\tau\to 0}\frac{\langle\Delta v\rangle}{\Delta\tau}, \quad B(v)=\lim_{\Delta\tau\to 0}\frac{\langle\Delta v^2\rangle}{\Delta\tau}.$$

From Eq. (6)

$$\Delta v \simeq (x-\sin\theta - v)\Delta\tau + \int_\tau^{\tau+\Delta\tau} d\tau\, \mathcal{I}(\tau),$$

so that

$$\langle\Delta v\rangle = (x-\sin\theta-v)\Delta\tau,$$

$$\langle\Delta v^2\rangle = (x-\sin\theta-v)^2\Delta\tau^2 + \int_\tau^{\tau+\Delta\tau} d\tau' \int_\tau^{\tau+\Delta\tau} d\tau'' \langle \mathcal{I}(\tau')\mathcal{I}(\tau'')\rangle .$$

Using Eq. (7) we thus find that

$$A(v)=x-\sin\theta-v, \quad B(v)=4/\gamma\beta_c ,$$

which lead to the Fokker-Planck equation

$$\frac{\partial P}{\partial\tau}=-\beta_c v\frac{\partial P}{\partial\theta}-(x-\sin\theta)\frac{\partial P}{\partial v}+\frac{\partial}{\partial v}(vP)+\frac{2}{\gamma\beta_c}\frac{\partial^2 P}{\partial v^2}. \quad (10)$$

Let us define the moments

$$g_n(\theta,\tau)=\int_{-\infty}^{+\infty} dv\, v^n P(v,\theta,\tau), \quad n=0,1,2\ldots, \quad (11)$$

so that the required average voltage is given by

$$\bar{v}=\frac{\int_0^{2\pi} d\theta \int_{-\infty}^{+\infty} dv\, vP}{\int_0^{2\pi} d\theta\int_{-\infty}^{+\infty} dv\, P}=\frac{\int_0^{2\pi} g_1 d\theta}{\int_0^{2\pi} g_0 d\theta}. \quad (12)$$

The moments g_n satisfy the following hierarchy of equations:

$$\frac{\partial g_0}{\partial\tau}=-\beta_c\frac{\partial g_1}{\partial\theta},$$

$$\frac{\partial g_1}{\partial\tau}=-\beta_c\frac{\partial g_2}{\partial\theta}-g_1+(x-\sin\theta)g_0, \quad (13)$$

$$\frac{\partial g_2}{\partial\tau}=-\beta_c\frac{\partial g_3}{\partial\theta}-2g_2+2(x-\sin\theta)g_1+\frac{4}{\gamma\beta_c}g_0,$$

$$\frac{\partial g_n}{\partial\tau}=-\beta_c\frac{\partial g_{n+1}}{\partial\theta}-ng_n+n(x-\sin\theta)g_{n-1}+\frac{2n(n-1)}{\gamma\beta_c}g_{n-2}.$$

More specifically, these equations imply that any steady-state solution must satisfy the conditions

$$g_1(\theta)=\text{const}\equiv g_1 \quad (14a)$$

and

$$-\tfrac{1}{2}\beta_c^2\frac{d^2 g_3}{d\theta^2}+(1-\beta_c\cos\theta)g_1-(x-\sin\theta)g_0+\frac{2}{\gamma}\frac{dg_0}{d\theta}=0. \quad (14b)$$

In the steady state, note that the condition (14a) leads to

$$\bar{v}=2\pi g_1\Big/\int_0^{2\pi} d\theta\, g_0(\theta). \quad (15)$$

III. SOLUTIONS OF THE FOKKER-PLANCK EQUATION

It is obvious from Eq. (13) that the hierarchy of equations for g_n cannot be decoupled unless $\beta_c=0$. One may argue, however, that one should truncate these equations, even if $\beta_c\neq 0$, by approximating g_3 in terms of g_2 etc., since the original Fokker-Planck equation is correct only up to terms of the order Δv^2. For example, one could write $g_3 \simeq (x-\sin\theta)g_2$ corresponding to the case when $\beta_c\to 0$ and $\gamma\beta_c\to\infty$, or $g_3\simeq(x-\sin\theta)g_2+(4/\gamma\beta_c)g_1$ corresponding to the case $\beta_c\to 0$, $\gamma\beta_c$ finite. However, since β_c occurs as a factor in the term with the highest derivative in θ in any given equation, the nature of the solutions will be different whether we put β_c identically equal to zero or not.

A. The Case with $\beta_c = 0$

If β_c is identically zero, from the exact Eq. (14b) for the steady state we obtain

$$\frac{dg_0}{d\theta}-\tfrac{1}{2}\gamma(x-\sin\theta)g_0=-\tfrac{1}{2}\gamma g_1. \quad (16)$$

This is nothing but the steady-state version of Smoluchowski's equation solved in Ref. 3. The periodic solution can be written in a convenient form

$$g_0(\theta)=\tfrac{1}{4}\gamma g_1(\sinh\tfrac{1}{2}\pi\gamma x)^{-1} f(\theta)\int_{-\pi}^{\pi} d\theta' f(\theta'-\theta), \quad (17)$$

where

$$f(\theta)=\exp[\tfrac{1}{2}\gamma(x\theta+\cos\theta)]. \quad (18)$$

This is equivalent to Eq. (8) of Ref. 3. Using Eq. (15) we obtain

$$\bar{v}=\frac{8\pi}{\gamma}\sinh\tfrac{1}{2}\pi\gamma x\left[\int_{-\pi}^{\pi} d\theta\int_{-\pi}^{\pi} d\theta' f(\theta)f(\theta'-\theta)\right]^{-1}, \quad (19)$$

which may be integrated to give

$$\bar{v}=x\left[\sum_{K=0}^{\infty}\epsilon_K(-1)^K I_K^2(\tfrac{1}{2}\gamma)\left(\frac{x^2\gamma^2}{x^2\gamma^2+4K^2}\right)\right]^{-1}, \quad (20)$$

where $\epsilon_K=1$ if $K=0$, $\epsilon_K=2$ if $K\neq 0$, and where I_K is the modified Bessel function of the first kind. Using the relations

$$I_K(Z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta\, e^{Z\cos\theta}\cos K\theta\ , \qquad (21a)$$

$$\sum_{K=0}^{\infty}(-1)^K \epsilon_K \frac{\cos K\theta}{K^2+x^2Z^2} = \frac{\pi\cosh\theta xZ}{xZ\sinh\pi xZ}\ , \quad -\pi<\theta<\pi \qquad (21b)$$

we can evaluate the asymptotic value of the above integral for $\gamma \gg 1$, in the region where $x<1$, by the method of steepest descent. One finds that

$$\bar{v} = 2(1-x^2)^{1/2}\sinh\tfrac{1}{2}\pi\gamma x \exp\{-\gamma[(1-x^2)^{1/2}+x\sin^{-1}x]\},$$
$$x<1,\ \gamma\gg 1\ . \qquad (22)$$

For $\beta_c=0$ and $\gamma\to\infty$ $(T\to 0)$ it should be noted directly from Eq. (14b) that $(x-\sin\theta)g_0=g_1$, which gives the well-known result

$$\bar{v}=0\,,\qquad x<1\quad(\gamma\to\infty)$$
$$=(x^2-1)^{1/2},\quad x>1\quad(\gamma\to\infty)\ . \qquad (23)$$

B. The Case with $\beta_c \neq 0$

In order to consider the case of arbitrary β_c, one has to go back to the original time-dependent Fokker-Planck Eq. (10). In general it is difficult to solve this equation for an arbitrary potential. This may however be solved for an oscillator. For $x<1$, our potential $u=-(x\theta+\cos\theta)$ has minima at $\theta_n^{\min}=\sin^{-1}x+2n\pi$ and maxima at $\theta_n^{\max}=\pi-\sin^{-1}x+2n\pi$. At zero temperature the system is expected to be in one of the metastable states given by $\theta_n^{\min}$. At a finite temperature the fluctuations can cause the system to go from one metastable state to another. When the temperature is not very high, i.e., if $1/\gamma \ll |u_{\max}-u_{\min}|\sim 1$, one may assume that the transitions of the system can be considered as discrete random walks.[5] The system can be assumed to stay in one of the local minima for a sufficiently long time to allow a thermal equilibrium solution to be used. The transition rate (escape rate) from one minimum to another can be calculated by finding the solution of the steady-state Fokker-Planck equation near the intermediate maximum.

Let q and p be the transition probability for the system to go from the state $n+1$ to n and from $n-1$ to n, respectively, so that $1-q-p$ is the probability of remaining in the state n itself. Then the probability $W(m|n;\ s+1)$ for the system to go from the state m to n in $s+1$ steps satisfies the equations

$$W(m|n,s+1)=(1-q-p)W(m|n,s)+qW(m|n+1,s)$$
$$+pW(m|n-1,s), \qquad (24)$$

$$\textstyle\sum_n W(m|n,s+1)=1\ , \qquad (25)$$

so that

$$\bar{v}=\frac{1}{\beta_c}\left\langle\frac{d\theta}{d\tau}\right\rangle$$
$$=\frac{1}{\beta_c}\textstyle\sum_n \theta_n^{\min}\displaystyle\frac{W(m|n,s+1)-W(m|n,s)}{\Delta\tau}$$
$$=\frac{1}{\beta_c}\textstyle\sum_n\displaystyle\left(\frac{\theta_{n+1}^{\min}-\theta_n^{\min}}{\Delta\tau}\cdot p-\frac{\theta_n^{\min}-\theta_{n-1}^{\min}}{\Delta\tau}q\right)W(m|n,s)$$
$$=\frac{2\pi}{\beta_c}(p'-q')\ , \qquad (26)$$

where p' and q' are the respective transition rates which are supposed to be independent of n.

Near the nth maximum, one can approximate the potential by an oscillator,

$$u\simeq u_n^{\max}+\cos\theta_n^{\max}\tfrac{1}{2}(\theta-\theta_n^{\max})^2$$
$$=u_n^{\max}-(\omega^2/\beta_c)\tfrac{1}{2}X^2\ , \qquad (27)$$

where

$$u_n^{\max}=-[x(\pi-\sin^{-1}x+2n\pi)-(1-x^2)^{1/2}]\ , \qquad (28)$$

$$X=\theta-\theta_n^{\max}\ , \qquad (29)$$

with the frequency[8]

$$\omega=+\beta_c^{1/2}(1-x^2)^{1/4}\ . \qquad (30)$$

Similarly, near the nth minimum

$$u\simeq u_n^{\min}+\tfrac{1}{2}(\omega^2/\beta_c)Y^2\ , \qquad (31)$$

where

$$u_n^{\min}=-[x(\sin^{-1}x+2n\pi)+(1-x^2)^{1/2}]\ , \qquad (32)$$

$$Y=\theta-\theta_n^{\min}\ . \qquad (33)$$

It is easy to verify that the Maxwell-Boltzmann distribution

$$P_0=Ce^{(-\gamma/2)(\beta_c v^2/2+u)} \qquad (34)$$

identically satisfies the Fokker-Planck equation (10). However, this cannot be the stationary solution in our case which will be valid for all θ. Only near a minimum (see Fig. 1) is this expected to be a valid solution, so that

$$P_{A_n}\simeq Ce^{-\gamma u_n^{\min}/2}e^{(-\gamma/2)(\beta_c v^2/2+\omega^2Y^2/2\beta_c)} \qquad (35)$$

In the vicinity of the nth maximum, we expect P to be of the form[9]

$$P_{C_n}\simeq CF(X,v)\exp(-\tfrac{1}{2}\gamma u_n^{\max})$$
$$\times\exp\{-\tfrac{1}{2}\gamma\,[\tfrac{1}{2}\beta_c v^2-\tfrac{1}{2}(\omega^2/\beta_c)X^2\},\qquad (36)$$

with

$$F(X,v)\to 1\quad\text{for } X\to-\infty$$
$$\to 0\quad\text{for } X\to+\infty\ . \qquad (37)$$

In the steady state, the equation satisfied by $F(X,v)$ can be obtained from Eqs. (35) and (10). One finds that

$$v\frac{\partial F}{\partial X}+\frac{\omega^2}{\beta_c}X\frac{\partial F}{\partial v}=-\frac{v}{\beta_c}\frac{\partial F}{\partial v}+\frac{2}{\gamma\beta_c^2}\frac{\partial^2 F}{\partial v^2}\ . \quad (38)$$

The required solution[9] of this equation is

$$F(X,v)=\left(\frac{\gamma\beta_c}{4\pi}\right)^{1/2}\left[\left(\tfrac{1}{4}+\omega^2\right)^{1/2}-\tfrac{1}{2}\right]^{1/2}$$
$$\times\int_{-\infty}^{v-(1/\beta_c)[(1/4+\omega^2)^{1/2}+1/2]}d\xi$$
$$\times\exp\left\{-\tfrac{1}{4}\gamma\beta_c\left[\left(\tfrac{1}{4}+\omega^2\right)^{1/2}-\tfrac{1}{2}\right]\xi^2\right\}\ . \quad (39)$$

Thus the rate of transition across C_n from $\theta_n^{\min}$ to $\theta_{n+1}^{\min}$ is

$$R_{n\to n+1}\simeq\frac{\int_{-\infty}^{\infty}dv\,P_{C_n}(X=0,v)\beta_c v}{\int_{-\infty}^{\infty}dv\int_{-\infty}^{\infty}dY P_{A_n}(Y,v)}$$
$$=\frac{1}{2\pi}\left[\left(\tfrac{1}{4}+\omega^2\right)^{1/2}-\tfrac{1}{2}\right]e^{(-\gamma/2)(u_n^{\max}-u_n^{\min})}\ . \quad (40)$$

We, therefore, obtain

$$p'-q'=\frac{1}{2\pi}\left[\left(\tfrac{1}{4}+\omega^2\right)^{1/2}-\tfrac{1}{2}\right]$$
$$\times\left\{\exp\left[-\tfrac{1}{2}\gamma\left(u_{n-1}^{\max}-u_{n-1}^{\min}\right)\right]-\exp\left[-\tfrac{1}{2}\gamma\left(u_n^{\max}-u_{n+1}^{\min}\right)\right]\right\}, \quad (41)$$

so that

$$\bar{v}=\frac{1}{\beta_c}\left\{\left[1+4\beta_c\left(1-x^2\right)^{1/2}\right]^{1/2}-1\right\}$$
$$\times\exp\left\{-\gamma\left[\left(1-x^2\right)^{1/2}+x\sin^{-1}x\right]\right\}\sinh\tfrac{1}{2}\pi\gamma x\ . \quad (42)$$

Note that our expression reduces to

$$\bar{v}=2\left(1-x^2\right)^{1/2}\sinh\tfrac{1}{2}\pi\gamma x$$
$$\times\exp\left\{-\gamma\left[\left(1-x^2\right)^{1/2}+x\sin^{-1}x\right]\right\},\quad \beta_c\to 0 \quad (43)$$

$$\bar{v}=\frac{2}{\sqrt{\beta_c}}\left(1-x^2\right)^{1/4}\sinh\tfrac{1}{2}\pi\gamma x$$
$$\times\exp\left\{-\gamma\left[\left(1-x^2\right)^{1/2}+x\sin^{-1}x\right]\right\},\quad \beta_c\to\infty \quad (44)$$

which agree with the results derived by Ambegaokar and Halperin,[3] and Ivanchenko and Zil'berman,[5] respectively, in the region where $x<1$ and $\gamma\gg 1$.

IV. CONCLUSION

It is clear from the random-walk model considered in Sec. III that our expression (42) is valid only at low temperatures, and in the region $x<1$. This can give correct results only if $\gamma\gg(u_n^{\max}-u_n^{\min})^{-1}=[2(1-x^2)^{1/2}+2x(\sin^{-1}x-\frac{1}{2}\pi)]^{-1}$, which implies that depending on the value of γ, the value

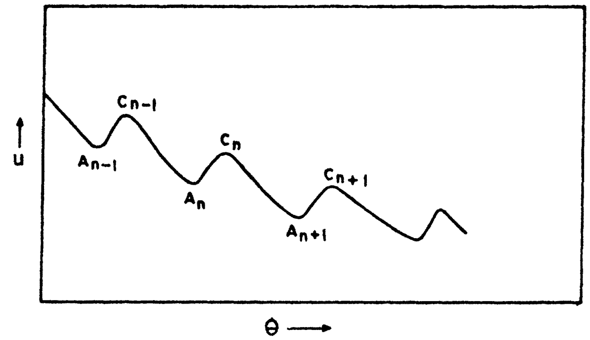

FIG. 1. Sketch of the potential u as a function of the phase difference θ for $I<I_1$.

of x should not be too close to 1. For a finite β_c, our solution is expected to be valid for x below $x_c=\alpha_c(\beta_c)$, where α_c is the cutoff current calculated by McCumber[2] in the absence of thermal fluctuations. Although all the experimental numbers are not available to us, it is obvious that in the region of small x or $\bar{v}$, the experimental data should be fitted with our expression (40) instead of the expression (19).

Since for large γ Eq. (42) gives correct limiting values for $\beta_c\to 0$ as well as $\beta_c\to\infty$, and since the correct solution given by Eq. (19) or (20) valid for any γ is known for $\beta_c=0$, it is perhaps a better approximation to write

$$\bar{v}=\frac{\left\{\left[1+4\beta_c\left(1-x^2\right)^{1/2}\right]^{1/2}-1\right\}}{2\beta_c\left(1-x^2\right)^{1/2}}\left[\bar{v}\right]_{\beta_c=0}\ , \quad (45)$$

$$\bar{v}=\frac{x\left\{\left[1+4\beta_c\left(1-x^2\right)^{1/2}\right]^{1/2}-1\right\}}{2\beta_c\left(1-x^2\right)^{1/2}}$$
$$\times\left[\sum_{K=0}^{\infty}\epsilon_K(-1)^K I_K^2\left(\tfrac{1}{2}\gamma\right)\frac{x^2\gamma^2}{x^2\gamma^2+4K^2}\right]^{-1} \quad (46)$$

in the region $x<1$. The solution for $x\geqslant 1$ could be taken to be the same as in the case of $\beta_c=0$.

It should be emphasized that the basic physical assumptions in obtaining our Eq. (42) are that the system spends most of its time in the valleys, and that it is almost everywhere in equilibrium. For this to happen,[10] the dissipative mechanism must be able to get rid of the energy kT or $x\hbar I_1/2e$, whichever is larger, before the system traverses a fraction of one valley. The exact conditions of the validity of Eq. (42) are thus $\gamma\gg 1$ and $x\ll x_c$. In the very small $-x$ limit the formula shows that the slope $d\bar{x}/d\bar{v}$ is larger for larger β_c at fixed γ. This is also indicated from numerical results of Kurkijarvi and Ambegaokar,[11] who solved Eqs. (1)–(3) by a Monte Carlo procedure, if their curves are properly extrapolated.

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[6]More precisely, the dimensionless parameter $\beta = (2eI_1/\hbar C)\, R^2C^2$, as defined in the text, is taken to be zero in Ref. 3, whereas the calculation in Ref. 5 is valid for $\beta_c \gg 1$ and $\gamma = (\hbar I_1/ekT) \gg 1$, in the region where $x = (I/I_1) < 1$. Although the experimental values of γ and β_c in Ref. 1 are obtained as a result of a two-parameter fit (I_1 and T) with the theoretical expression due to Ambegaokar and Halperin, and hence may not be relevant for a correct theory, their orders of magnitude should be correct. These values for β_c range from 1 to 4, and γ varies between 3 and 15.
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Superconductivity of Dilute Indium-Thallium Alloys*

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Critical-field measurements, determined from isothermal magnetization curves, are reported for InTl alloys covering the concentration range of 0–7% Tl. Since In and Tl have the same number of valence electrons, the valence effects which tend to mask the effect of anisotropy in the energy gap should be minimized. Critical-field measurements down to $T = 0.35\ ^\circ$K provide an accurate determination of H_0, the critical field at $T=0$, and of γ, the temperature coefficient of the normal electronic specific heat. New values for γ (=1.672 ± 0.003 mJ/mole °K) and for H_0 (=281.53 ± 0.06 G) are reported for pure In. No effect on γ due to alloying is observed; however, changes in the quantities T_c, H_0/T_c, and $(dH_c/dT)_{T=T_c}$ are observed and are compared with the anisotropy theory of Markowitz and Kadanoff and of Clem. Significant departures from the predictions of anisotropy theory are noted for all the measured parameters.

I. INTRODUCTION

The effects of dilute nonmagnetic impurities on the superconducting properties of pure metals have been studied experimentally for some time.[1–5] These effects are separated into two classes to distinguish those which do from those which do not depend on the particular type of impurity. The impurity-independent effects are attributed to changes in the isotropic mean free path (IMFP) of an electron[6,7] and to changes in the anisotropy of the superconducting energy gap, Δ_k.[8–10] The impurity-dependent effects, the so-called valence effects, are attributed to changes in the basic parameters of the metal, such as the electronic density of states. Anisotropy effects on the thermodynamic properties of superconductors are typically much smaller than valence effects and are therefore difficult to isolate and compare with theoretical predictions.

Valence effects are minimal in InTl alloys since In and Tl have the same number of valence electrons. The electron concentration n in this alloy system is constant, and all quantities dependent on n, such as the electronic density of states, should not be affected by the additions of Tl. Therefore, InTl alloys should be particularly suitable for studying anisotropy-induced changes in superconducting properties.

II. EXPERIMENTAL TECHNIQUE

A. Apparatus

The apparatus used to measure the critical fields and critical temperatures of the samples is shown in Fig. 1. The samples were positioned inside the cryostat by supporting them in a copper sample holder which was attached to the end of a long sample rod (see Fig. 1). This sample rod could be removed to change the sample and replaced within the cryostat while the system remained at cryogenic temperatures.

For measurements below 1°K, the superconducting sample was surrounded by a bath of liquid He^3 whose temperature was controlled by regulating the